CSE 130 : Programming Languages

Higher-Order Functions

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Recursion

- A way of life
- A different way to view computation
  - Solutions for bigger problems
  - From solutions for sub-problems

Why know about it ?

1. Often far simpler, cleaner than loops
   - But not always…
2. Forces you to factor code into reusable units
   - Only way to "reuse" loop is via cut-paste

Q: What does this evaluate to ?

```
let rec foo i j =
  if i >= j then []
  else i::(foo (i+1) j)
in foo 0 3
```

(a) `[0;1;2]`

(b) `[0;0;0]`

(c) `[]`

(d) `[2;2;2]`

(e) `[2;1;0]`

Q: What does this evaluate to ?

```
let rec range i j =
  if i >= j then []
  else i::(range (i+1) j)
```

range 3 3 ====> []

range 2 3 ====> 2::(range 3 3) ====> 2::[]

range 1 3 ====> 1::(range 2 3) ====> 1::2::[]

range 0 3 ====> 0::(range 1 3) ====> 0::1::2::[]

Q: What does this evaluate to ?

```
let rec range i j =
  if i >= j then []
  else i::(range (i+1) j)
```

Tail Recursive?

Q: What does this evaluate to ?

```
let range lo hi =
  let rec helper res j =
    if lo >= j then res
    else helper (j::res)(j-1)
  in helper [] hi
```

Tail Recursive!

Moral of the day…

Recursion good…

…but HOFS better!

News

- PA2 due **FRIDAY @ 23:59:59pm**
- PA3 goes up soon
- Midterm **Monday 5/2**
  - In class
  - Open book etc.
  - Practice materials on webpage

Today's Plan

- A little more practice with recursion
  - Base Pattern -> Base Expression
  - Induction Pattern -> Induction Expression

- Higher-Order Functions
  - or, why "take" and "return" functions ?

Write: evens

```
(* val evens: int list -> int list *)
let rec evens xs = match xs with
   | []        -> ...
   | x::xs'    -> ...
```

evens [] ====> []

evens [1;2;3;4] ====> [2;4]

Write: evens

```
(* val evens: int list -> int list *)
let rec evens xs = match xs with
  | []      -> []
  | x::xs'  -> if x mod 2 = 0
               then x::(evens xs')
               else (evens xs')
```

evens []   ====> []

evens [1;2;3;4] ====> [2;4]

Write: fourLetters

```
(* fourLetters: string list -> string list *)
let rec fourLetters xs = match xs with
  | []     -> ...
  | x::xs’ -> ...
```

fourLetters []

====> []

fourLetters [“cat”;“must”;“do”;“work”]

====> [“must”; “work”]

Write: evens

```
(* fourLetters: string list -> string list *)
let rec fourLetters xs = match xs with
   | []      -> []
   | x::xs'  -> if length x = 4
                then x::(fourLetters xs')
                else (fourLetters xs')
```

fourLetters []

====> []

fourLetters [“cat”;“must”;“do”;“work”]

====> [“must”; “work”]

```
(* val evens: int list -> int list *)
let rec evens xs = match xs with
  | []       -> []
  | x::xs’  -> if x mod 2 = 0
               then x::(evens xs’)
               else (evens xs’)
```

```
(* fourLetters: string list -> string list *)
let rec fourLetters xs = match xs with
  | []       -> []
  | x::xs’  -> if length x = 4
               then x::(fourLetters xs’)
               else (fourLetters xs’)
```

Yuck! Most code is same!

```
(* evens: int list -> int list *)
let rec foo xs = match xs with
  | []      -> []
  | x::xs'  -> if x mod 2 = 0
               then x::(foo xs')
               else (foo xs')
```

```
(* fourLetters: string list -> string list *)
let rec foo xs = match xs with
  | []      -> []
  | x::xs'  -> if length x = 4
               then x::(foo xs')
               else (foo xs')
```

Yuck! Most code is same!

Moral of the Day...

“D.R.Y”

Don’t Repeat Yourself!

Moral of the Day…

HOFs Allow “Factoring”

General “Pattern”

+

Specific “Operation”

```
let rec evens xs =
  match xs with
  | []       -> []
  | x::xs’ -> if x mod 2 = 0
              then x::(foo xs’)
              else (foo xs’)
```

```
letrec fourLetters xs =
  match xs with
  | []       -> []
  | x::xs’ -> if length x = 4
              then x::(foo xs’)
              else (foo xs’)
```

```
let rec filter f xs =
  match xs with
  | []       -> []
  | x::xs’ -> if f x
              then x::(filter xs’)
              else (filter xs’)
```

The “filter” pattern

```
let rec evens xs =
  match xs with
  | []      -> []
  | x::xs’  -> if x mod 2 = 0
               then x::(foo xs’)
               else (foo xs’)
```

```
let rec fourLetters xs =
  match xs with
  | []      -> []
  | x::xs’  -> if length x = 4
               then x::(foo xs’)
               else (foo xs’)
```

```
let evens xs =
  filter (fun x -> x mod 2 = 0) xs
```

```
let fourLetters xs =
  filter (fun x -> length x = 4) xs
```

```
let rec filter f xs =
  match xs with
  | []      -> []
  | x::xs’  -> if f x
               then x::(filter xs’)
               else (filter xs’)
```

The “filter” pattern

Factor Into Generic + Specific

Specific Operations

```
let evens xs =
  filter (fun x -> x mod 2 = 0) xs
```

```
let fourLetters xs =
  filter (fun x -> length x = 4) xs
```

```
let rec filter f xs =
  match xs with
  | []      -> []
  | x::xs'  -> if f x
               then x::(filter xs')
               else (filter xs')
```

Generic “filter” pattern

Write: listUpper

```
(* string list -> string list *)
let rec listUpper xs =
  match xs with
  | []      -> ...
  | x::xs'-> ...
```

listUpper [] ====> []

listUpper [“carne”; “asada”] ====> [“CARNE”; “ASADA”]

Write: listUpper

```
(* string list -> string list *)
let rec listUpper xs =
  match xs with
   | []     ->[]
   | x::xs'->(uppercase x)::(listUpper xs')
```

listUpper [] ====> []

listUpper [“carne”; “asada”] ====> [“CARNE”; “ASADA”]

Write: listSquare

```
(* int list -> int list *)
let rec listSquare xs =
  match xs with
   | []      -> ...
   | x::xs'-> ...
```

listSquare []   ====> []

listSquare [1;2;3;4;5] ====> [1;4;9;16;25]

Write: listSquare

```
(* int list -> int list *)
let rec listSquare xs =
  match xs with
  | []     -> []
  | x::xs'-> (x*x)::(listSquare xs')
```

listSquare []   ====> []

listSquare [1;2;3;4;5] ====> [1;4;9;16;25]

Yuck! Most code is same!

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs'->(uppercase x)::(listUpper xs')
```

```
let rec listSquare xs =
  match xs with
  | []     -> []
  | x::xs'-> (x*x)::(listSquare xs')
```

What’s the Pattern?

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs'->(uppercase x)::(listUpper xs')
```

```
let rec listSquare xs =
  match xs with
  | []     -> []
  | x::xs'-> (x*x)::(listSquare xs')
```

What's the Pattern?

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs'->(uppercase x)::(listUpper xs')
```

```
let rec listSquare xs =
  match xs with
  | []     -> []
  | x::xs'-> (x*x)::(listSquare xs')
```

“Refactor” Pattern

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs’->(uppercase x)::(listUpper xs’)
```

```
let rec listSquare xs =
  match xs with
  | []     -> []
  | x::xs’-> (x*x)::(listSquare xs’)
```

```
let rec pattern ...
```

“Refactor” Pattern

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs’->(uppercase x)::(listUpper xs’)
```

```
let rec listSquare xs =
  match xs with
  | []     -> []
  | x::xs’-> (x*x)::(listSquare xs’)
```

```
let rec map f xs =
  match xs with
  | []     -> []
  | x::xs’-> (f x)::(map f xs’)
```

"Refactor" Pattern

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs'->(uppercase x)::(listUpper xs')
```

```
let listUpper xs = map (fun x -> uppercase x) xs
```

```
let rec map f xs =
  match xs with
  | []     -> []
  | x::xs'-> (f x)::(map f xs')
```

"Refactor" Pattern

```
let rec listUpper xs =
  match xs with
  | []     ->[]
  | x::xs'->(uppercase x)::(listUpper xs')
```

```
let listUpper = map uppercase
```

```
let rec map f xs =
  match xs with
  | []     -> []
  | x::xs'-> (f x)::(map f xs')
```

“Refactor” Pattern

```
let listSquare = map (fun x -> x*x)
```

```
let rec listSquare xs =
  match xs with
  | []      -> []
  | x::xs’-> (x*x)::(listSquare xs’)
```

```
let rec map f xs =
  match xs with
  | []      -> []
  | x::xs’-> (f x)::(map f xs’)
```

Factor Into Generic + Specific

```
let listSquare = map (fun x -> x * x)
```

```
let listUpper  = map uppercase
```

Specific Op

```
let rec map f xs =
  match xs with
  | []     -> []
  | x::xs'-> (f x)::(map f xs')
```

Generic "iteration" pattern

Moral of the Day…

“D.R.Y”

Don’t Repeat Yourself!

Q: What is the type of map?

```
let rec map f xs =
  match xs with
  | []      -> []
  | x::xs'  -> (f x)::(map f xs')
```

(a) ``(`a -> `b) -> `a list -> `b list``

(b) `(int -> int) -> int list -> int list`

(c) `(string -> string) -> string list -> string list`

(d) ``(`a -> `a) -> `a list -> `a list``

(e) ``(`a -> `b) -> `c list -> `d list``

Q: What is the type of map?

```
let rec map f xs =
  match xs with
  | []       -> []
  | x::xs’   -> (f x)::(map f xs’)
```

(a) `(`a -> `b) -> `a list -> `b list`

Type says it all !

- Apply “f” to each element in input list
- Return a list of the results

Q: What does this evaluate to ?

```
map (fun (x,y) -> x+y) [1;2;3]
```

(a) **`[2;4;6]`**

(b) **`[3;5]`**

(c) Syntax Error

(e) Type Error

Don’t Repeat Yourself!

```
let rec map f xs =
  match xs with
  | []         -> []
  | x::xs’  -> (f x)::(map f xs’)
```

“Factored” code:

- Reuse iteration template
- Avoid bugs due to repetition
- Fix bug in one place !

Don’t Repeat Yourself!

```
let rec map f xs =
  match xs with
  | []        -> []
  | x::xs'   -> (f x)::(map f xs')
```

Made Possible by **Higher-Order Functions!**

Recall: len

```
(* 'a list -> int *)
let rec len xs =
  match xs with
   | []     -> 0
   | x::xs'-> 1 + len xs'
```

len [] ====> 0

len [“carne”; “asada”] ====> 2

Recall: sum

```
(* int list -> int *)
let rec sum xs =
  match xs with
   | []      -> 0
   | x::xs'-> x + sum xs'
```

sum []   ====> 0

sum [10;20;30] ====> 60

Write: concat

```
(* string list -> string *)
let rec concat xs =
  match xs with
  | []      -> ""
  | x::xs'-> x ^ (concat xs')
```

concat []

====> “”

concat [“carne”; “asada”; “torta”]

====> “carneasadatorta”

Write: concat

```
(* string list -> string *)
let rec concat xs =
  match xs with
  | []     -> ""
  | x::xs'-> x^(concat xs')
```

concat []

====> “”

concat [“carne”; “asada”; “torta”]

====> “carneasadatorta”

What’s the Pattern?

```
let rec len xs =
  match xs with
  | []     -> 0
  | x::xs’-> 1 + (len xs’)
```

```
let rec sum xs =
  match xs with
  | []     -> 0
  | x::xs’-> x + (sum xs’)
```

```
let rec concat xs =
  match xs with
  | []     -> “”
  | x::xs’-> x^(concat xs’)
```

What’s the Pattern?

```
let rec len xs =
  match xs with
  | []     -> 0
  | x::xs’-> 1 + (len xs’)
```

```
let rec sum xs =
  match xs with
  | []     -> 0
  | x::xs’-> x + (sum xs’)
```

```
let rec foldr f b xs =
  match xs with
  |[]     -> b
  |x::xs’-> f x (foldr f b xs’)
```

```
let rec concat xs =
  match xs with
  | []     -> “”
  | x::xs’-> x^(concat xs’)
```

```
let rec foldr f b xs =
  match xs with
  |[]     -> b
  |x::xs'-> f x (foldr f b xs')
```

```
let rec len xs =
  match xs with
  | []      -> 0
  | x::xs'-> 1 + (len xs')
```

```
let len =
  foldr (fun x n -> n+1) 0
```

```
let rec sum xs =
  match xs with
  | []      -> 0
  | x::xs'-> x + (sum xs')
```

```
let sum =
  foldr (fun x n -> x+n) 0
```

```
let rec concat xs =
  match xs with
  | []      -> ""
  | x::xs'-> x^(concat xs')
```

```
let concat =
  foldr (fun x n -> x^n) ""
```

“fold” Pattern

```
let rec foldr f b xs =
  match xs with
  |[]     -> b
  |x::xs’-> f x (foldr f b xs’)
```

```
let len =
  foldr (fun x n -> n+1) 0
```

```
let sum =
  foldr (fun x n -> x+n) 0
```

```
let concat =
  foldr (fun x n -> x^n) ””
```

Specific Op

Q: What does this evaluate to ?

```
foldr (fun x n -> x::n) [] [1;2;3]
```

```
let rec foldr f b xs =
  match xs with
  |[]     -> b
  |x::xs’-> f x (foldr f b xs’)
```

(a) **[1;2;3]**

(b) **[3;2;1]**

(c) **[]**

(d) **[[3];[2];[1]]**

(e) **[[1];[2];[3]]**

“fold-right” pattern

```
let rec foldr f b xs =
   match xs with
    |[]      -> b
    |x::xs'-> f x (foldr f b xs')
```

foldr f b [x1;x2;x3]

====> f x1 (foldr f b [x2;x3])

====> f x1 (f x2 (foldr f b [x3]))

====> f x1 (f x2 (f x3 (foldr f b [])))

====> f x1 (f x2 (f x3 (foldr f b [])))

====> f x1 (f x2 (f x3 (b)))

The "fold" Pattern

```
let rec foldr f b xs =
  match xs with
   |[]     -> b
   |x::xs'-> f x (foldr f b xs')
```

Tail Recursive?

The “fold” Pattern

```
let rec foldr f b xs =
  match xs with
   |[]     -> b
   |x::xs’-> f x (foldr f b xs’)
```

Tail Recursive?

No!

Write: concat (TR)

```
let concat xs = ...
```

concat []

====> “”

concat [“carne”; “asada”; “torta”]

====> “carneasadatorta”

Write: concat

```
let concat xs =
  let rec helper res = function
  | []     -> res
  | x::xs’-> helper (res^x) xs’
in helper “” xs
```

helper “” [“carne”; “asada”; “torta”]

====> helper “carne” [“asada”; “torta”]

====> helper “carneasada” [“torta”]

====> helper “carneasadatorta” []

====> “carneasadatorta”

Write: sum (TR)

```
let sum xs = ...
```

sum [] ====> 0

sum [10;20;30] ====> 60

Write: concat

```
let sum xs =
  let rec helper res = function
  | []      -> res
  | x::xs'-> helper (res+x) xs'
in helper 0 xs
```

helper 0 [10; 100; 1000]

====> helper 10 [100; 1000]

====> helper 110 [1000]

====> helper 1110 []

====> 1110

What's the Pattern?

```
let sum xs =
  let rec helper res = function
  | []      -> res
  | x::xs'-> helper (res + x) xs'
in helper 0 xs
```

```
let concat xs =
  let rec helper res = function
  | []      -> res
  | x::xs'-> helper (res ^ x) xs'
in helper "" xs
```

```
let sum xs =
  foldl (fun res x -> res + x) 0
```

```
let sum xs =
  foldl (fun res x -> res ^ x) ""
```

```
let foldl f b xs =
  let rec helper res = function
  | []      -> res
  | x::xs'-> helper (f res x) xs'
in helper b xs
```

“Accumulation” Pattern

```
let foldl f b xs =
  let rec helper res = function
  | []      -> res
  | x::xs’-> helper (f res x) xs’
in helper b xs
```

```
let sum xs =
  foldl (fun res x -> res + x) 0
```

```
let sum xs =
  foldl (fun res x -> res ^ x) “”
```

Specific Op

Q: What does this evaluate to ?

```
foldl (fun res x -> x::res) [] [1;2;3]
```

```
let foldl f b xs =
  let rec helper res xs = match xs with
  | []      -> res
  | x::xs'-> helper (f res x) xs'
in helper b xs
```

(a) [1;2;3]

(b) [3;2;1]

(c) []

(d) [[3];[2];[1]]

(e) [[1];[2];[3]]

Funcs taking/returning funcs

Identify common computation "patterns"

- Filter values in a set, list, tree …
- Iterate a function over a set, list, tree …
  map
- Accumulate some value over a collection
  fold

Pull out (factor) "common" code:

- Computation Patterns
- Re-use in many different situations

Another fun function: “pipe”

```
let pipe x f = f x
```

```
let (|>) x f = f x
```

Compute the sum of squares of numbers in a list ?

```
let sumOfSquares xs =
    xs |> map (fun x -> x * x)
       |> foldl (+) 0
```

Tail Rec ?

Funcs taking/returning funcs

Identify common computation "patterns"

- Filter values in a set, list, tree …
- Convert a function over a set, list, tree … **map**
- Iterate a function over a set, list, tree …
- Accumulate some value over a collection **fold**

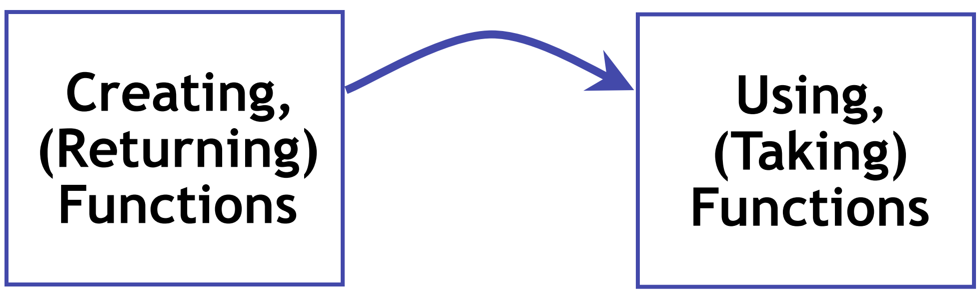
Pull out (factor) "common" code:

- Computation Patterns
- Re-use in many different situations

Functions are “first-class” values

- Arguments, return values, bindings …
- What are the benefits ?

Parameterized,
similar functions
(e.g. Testers)



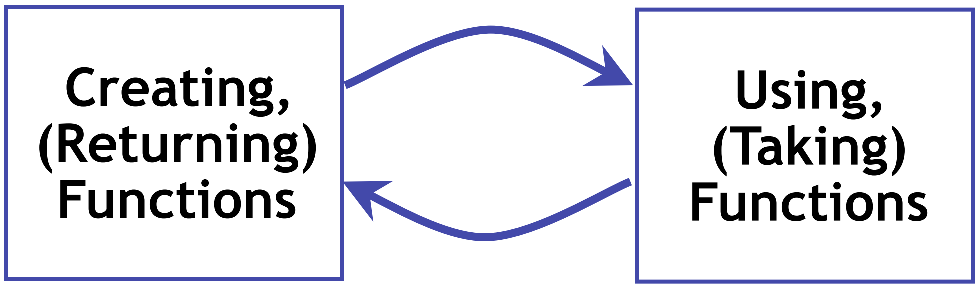
Iterator, Accumul,
Reuse computation
pattern w/o
exposing local info

Functions are "first-class" values

- Arguments, return values, bindings …
- What are the benefits ?

Parameterized, similar functions (e.g. Testers)

Iterator, Accumul, Reuse computation pattern w/o exposing local info

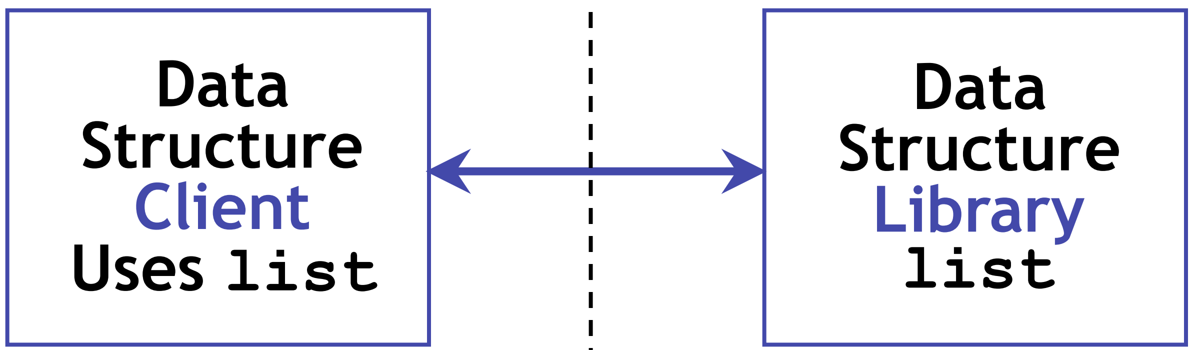


Compose Functions: Flexible way to build Complex functions from primitives.

Funcs taking/returning funcs

Higher-order funcs enable modular code

- Each part only needs local information



Uses meta-functions:
`map`,`fold`,`filter`
With locally-dependent funs
`(lt h)`, `square` etc.
Without requiring Implement. details of data structure

Provides meta-functions:
`map`,`fold`,`filter`
to traverse, accumulate over lists, trees etc.
Meta-functions don't need client info (tester ? accumulator ?)